**Topics: Normal distribution, Functions of Random Variables**

1. **The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**
2. **0.3875**
3. **0.2676**
4. **0.5**
5. **0.6987**

Ans:

0.2676

1. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.**
2. **More employees at the processing center are older than 44 than between 38 and 44.**

**Ans:**

True

We have a normal distribution with *μ* = 38 and *σ* = 6. Let X be the number of employees. So according to question

Probability of employess greater than age of 44 = pr(X>44)

Pr(X>44) = 1-pr(X44)

Z = (X -*μ*)/ = (X - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6) = Pr(Z ≤ 1)=84.1345%

Probability that the employee will be greater than age of 44 = 100-84.1345=15.86%

So, the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%

Therefore, the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

1. **A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

**Ans:**

True

Probability of employes less than age of 30 = pr(x<30).

Z = (X -*μ*)/ = (30 - 38)/6

Thus the question can be answered by using the normial table to find

Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12%

So, the number of employess with probability 00.912 of them being under age 30 = 0.0912\*400

Therefore, the statement B of the question is also TRUE.

1. **If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.**

**Ans:**

As we know that if X ~ N (μ1, σ1^2), and Y ∼ N (μ2, σ2^2) are two independent random variables then X + Y ~ N(μ1 + μ2, σ1^2 + σ2^2), and X − Y ∼ N (μ1 − μ2, σ1^2 + σ2^2).

Similarly if z = aX + bX, where and where X and Y are as defined above, i.e z id linear combination of X and Y , then Z ~ N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 ).

Therefore, in the question

2X1~ N (2 u,4 σ^2) and X1+X2~N (μ + μ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4μ,6 σ^2)

1. **Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**
2. **90.5, 105.9**
3. **80.2, 119.8**
4. **22, 78**
5. **48.5, 151.5**
6. **90.1, 109.9**

**Ans:**

D. 48.5,151.5

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The probability of getting value between a and b should be 0.99

So the probability of going wrong , or the probability outside the a and b area is 0.01 (l.e 1.99).

The probability towards left from a = -0.005(i.e 0.01/2)

The probability towards right from b = 0.005 (I.e 0.01/2)

So since we have the probabilies of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct**.**

1. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**
2. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**

**Ans;**

Range = 12-19.6\*5,12 +1.96\*5

= $2.2, $22.8

= Rs.99, Rs. 1026

1. **Specify the 5th percentile of profit (in Rupees) for the company**

**Ans:**

P(Z<=(p-12/5)=0.05

P-12/5 = -1.644

P = 12 – 8.22

=$3.78 or = Rs. 170.1

1. **Which of the two divisions has a larger probability of making a loss in a given year?**

**Ans:**

The first division of company , thus have large probability of making loss in agiven year.